# Mathematical \& Statistical Technique II 

## Module 1

1. Find the derivatives of the following functions:
a) $y=x-e^{x}+29$
b) $y=\left(e^{x}+2\right)\left(2 x^{2}+x+4\right)$
c) $y=4 x^{7}-\log x+\sqrt{ } x$
d) $y=(\log x+x)\left(5 x^{5}+55\right)$
e) $y=\left(x+e^{x}\right)(\log x-10)$
f) $y=6 x^{5}+\log 90+2\left(4^{x}\right)+e^{x}$
2. If $M R=55$ and $A R=98$, find $\eta$.
3. If $A R=65$ and $\eta=3$, find $M R$.
4. If $M R=85$ and $\eta=4.5$, find AR.
5. The cost of producing $x$ items is given by $2 x^{2}+5 x+20$. Find the total cost and marginal cost when $\mathrm{x}=10$.
6. The total cost function is $C=x^{3}-9 x^{2}+24 x+70$. Find $x$ for which the total cost is minimum.
7. The cost of producing $x$ items is given by $x^{3}+4 x+15$. Find the total cost and average cost when $x=6$
8. The total cost function is given by $C=x^{3}+2 x^{2}+5 x+30$. Find the total cost and marginal cost when $x=10$.
9. If the demand function is given by $D=15-4 p+p^{2}$, find the price elasticity of demand at $\mathrm{p}=1$.
10. A manufacturer can sell $\times$ items at a price of Rs. (330-x) each. The cost of producing $x$ items is Rs. $\left(x^{2}+10 x+12\right)$. Find $x$ for whieh the profit is maximum.
11. The total revenue function is given by $R=2 x^{3}-63 x^{2}+648 x+250$. Find $x$ for which the total revenue is maximum.

## Module 2

1. A principal amounts to Rs. 9680 after 3 years and to Rs. 10,800 after 5 years. Find the principal and the rate of simple interest.
2. Find the final amount of Rs. 10,000 at $9 \%$ p.a. in 3 years compounded half yearly.
3. Find the present value of Rs. 50,000 required after 3 years at $6 \%$ p.a. compounded annually.
4. What amount would be accumulated at the end of 3 years if an annuity of Rs. 20,000 is deposited at the end of each year? The rate of interest is $10 \%$ p.a. compounded annually.
5. Rajiv took a loan of Rs. 60,000 with $10 \%$ interest per month to be repaired in 5 months. Calculate the EMI using reducing balance method. Also calculate the interest and the principal repayment component for each EMI.
6. A sum of Rs. 50,000 accumulated to Rs. 82,000 after 8 years in a bank. Find the rate of simple interest which was charged by the bank.
7. Find the amount on maturity at the end of 2 years of Rs. 30,000 deposited at $10 \%$ p.a. compounded half yearly.
8. Diana deposited Rs. 1650 at the end of each quarter for $31 / 2$ years at $9 \%$ per annum compound interest. Find the amount she will receive at the end of the period.
9. Find the present value of Rs. 6000 payable 2 years hence if the interest is compounded annually at $8 \%$.
10. Manoj takes a loan of Rs. 80,000 to be repaid in 4 EMI at $12 \%$ per annum by reducing balance interest rate. Find the equated monthly instalments and also calculate the interest and the principal repayment component for each EMI.
11. At what rate will the simple interest on Rs. 15,000 for 4 years be equal to the simple interest on Rs. 16,000 for 3 years at $10 \%$ p.a.?
12. Find the amount accumulated after 6 years if a sum of Rs 25,000 is kept in a fixed deposit at a compound interest of $9 \%$ p.a.
13. Find the present worth of Rs. 14,641 at $10 \%$ rate of interest payable 4 years from now?
14. What is the accumulated value after 4 years on an immediate annuity of $\bar{R} s .8000 \overline{\mathrm{p}}$.a., the rate of interest being $8 \%$ per annum?
15. Radha purchases TV worth Rs. 5000 from a dealer at $10 \%$ p.a. Find the EMI if the repayment is to be done in 6 months. Also calculate the interest and principle repayment component for each EMI.

## Module 3

1. Calculate the Karl Pearson's correlation coefficient from the following

| X | 12 | 10 | 20 | 13 | 15 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Y | 7 | 14 | 6 | 12 | 11 |

2. Calculate the coefficient of rank correlation from the data given below:

| X | 54 | 61 | 44 | 32 | 24 | 12 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y | 64 | 25 | 15 | 36 | 40 | 56 |

3. Calculate the Karl Pearson's correlation coefficient from the following

| X | 18 | 12 | 16 | 14 | 10 | 15 | 17 | 13 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y | 9 | 13 | 20 | 15 | 11 | 24 | 26 | 22 |

4. Calculate the Spearman's Rank correlation coefficient from the following

| X | 40 | 33 | 60 | 59 | 50 | 55 | 48 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y | 70 | 60 | 85 | 75 | 72 | 82 | 69 |

5. Marks given by two Judges to a group of 10 participants are as follows. Calculate the coefficient of rank correlation

| X | 52 | 53 | 42 | 60 | 45 | 41 | 37 | 38 | 25 | 27 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y | 65 | 68 | 43 | 38 | 77 | 48 | 35 | 30 | 25 | 50 |

6. You are given the information about advertising expenditure and sales:

|  | Advt. expenses | Sales |
| :--- | :--- | :--- |
| Mean | 10 | 90 |
| SD | 3 | 12 |

Coefficient of correlation between sales and expenditure on Advertisement is 0.8. Find the likely sales when advertisement budget is Rs. 15 Lakh.
7. A sample of 50 students in a school gave the following statistics about Marks of students in Subjects of Mathematics and Science

|  | Maths | Science |
| :--- | :--- | :--- |
| Mean | 58 | 79 |
| SD | 12 | 18 |

Coefficient of correlation between the marks in Mathematics and marks in Science is 0.8. Approximate the marks of a student in the subject of Mathematics whose score in Science is 65 .
8. On the basis of the following information:

|  | $X$ | $Y$ |
| :--- | :--- | :--- |
| Mean | 40 | 45 |
| SD | 10 | 9 |

Karl Pearson's coefficient of correlation between $x$ and $y=0.50$. Also estimate the value of $x$ when $y=48$ using the appropriate equation.
9. Short note on Types of Correlation.
10. Short note on Scatter Diagram
11. Two random variables have the regression equations: $5 x+7 y-22=0$ and $6 x+2 y-20=0$. Find the mean values of $x$ and $y$ and coefficient of correlation.
12. Find the means values of $x$, $y$ and $r$ from the two regression equations. $3 x+2 y-26=0$ and $6 x+y-31=0$.
13. The two regression equations for a certain data were $y=x+5$ and $16 x=9 y-94$. Find values of $x, y$ and $r$.

## Module 4

1. Find trend values using 4 yearly moving averages for the following data.

| Year | 1980 | 1981 | 1982 | 1983 | 1984 | 1985 | 1986 | 1987 | 1988 | 1989 | 1990 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y | 242 | 250 | 253 | 249 | 254 | 256 | 250 | 257 | 262 | 268 | 260 |

2. Find 5 yearly moving average for the following data.

| Year | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y | 51 | 53 | 56 | 57 | 60 | 55 | 59 | 62 | 68 | 70 |

3. Find 3 yearly moving averages for the following time series giving Exports of a company.

| Year | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Exports | 46 | 53 | 72 | 57 | 62 | 78 | 60 | 85 |

4. Short note on components of time series.
5. Fit a straight line trend to the following data representing imports in million Rs. of a certain company. Also find an estimate for the year 2008.

| Year | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Imports | 48 | 50 | 58 | 52 | 45 | 41 | 49 |

6. Calculate Laspeyre's and Paasche's Price Index number for the following data:

| Commodity | $\mathrm{p}_{0}$ | $\mathrm{q}_{0}$ | $\mathrm{p}_{1}$ | $\mathrm{q}_{1}$ |
| :--- | :--- | :--- | :--- | :--- |
| A | 9 | 5 | 15 | 5 |
| B | 8 | 10 | 12 | 11 |
| C | 4 | 6 | 5 | 6 |
| D | 1 | 4 | 2 | 8 |

7. Calculate Fisher's Price Index number for the following data:

| Commodity | $\mathrm{p}_{0}$ | $\mathrm{q}_{0}$ | $\mathrm{p}_{1}$ | $\mathrm{q}_{1}$ |
| :--- | :--- | :--- | :--- | :--- |
| A | 9 | 5 | 15 | 5 |
| B | 8 | 10 | 12 | 11 |
| C | 4 | 6 | 5 | 6 |
| D | 1 | 4 | 2 | 8 |

8. Short note on components of Time Series.
9. Calculate the cost of living index number for the following data.

| Group | I | W |
| :--- | :--- | :--- |
| A | 48 | 160 |
| B | 7 | 120 |
| C | 10 | 140 |
| D | 10 | 100 |
| E | 15 | 80 |

10. Calculate the cost of living index number for the following data.

| Group | I | W |
| :--- | :--- | :--- |
| A | 180 | 30 |
| B | 145 | 15 |
| C | 150 | 10 |
| D | 170 | 5 |
| E | 190 | 5 |

11. Calculate the cost of living index number for the following data.

| Group | I | W |
| :--- | :--- | :--- |
| A | 221 | 35 |
| B | 198 | 14 |
| C | 190 | 15 |
| D | 183 | 8 |
| E | 161 | 20 |

## Module 5

1. Helpline cleans at $90 \%$ of the customers are given helpful stop if 10 customers are selected at random find probability that out of them the number of customers help is (i) exactly 6 (ii) 6 to 8 (including both)
2. The probability that the marriage will be broken within 3 years is $5 \%$. Find the probability that out of 60 married couples the number of marriages broken within 2 years is (i) more than one (ii) nil (Given that $\mathrm{e}^{-3}=0.0498$ )
3. State the properties of normal distribution
4. The probability that a student is a swimmer is $4 / 5$. Out of 5 students selected, find the probability that (i) 4 are swimmers (ii) one or less are swimmers
5. The weekly wages of 8000 workers are normally distributed with mean Rs. 770 and S.D. Rs. 70. Find the number of workers who wages below Rs. 700 (Area between $\mathrm{z}=0$ and z $=1$ is 0.3413 )
6. $30 \%$ of the students in the class are girls. Find the probability that are randomly selected group of 5 students include 3 girls.
7. A random variable $X$ follows poison distribution with mean $=2$. Find the probability of (i) 0 successes (ii) at most two successes (Given $\mathrm{e}^{-2}=0.135$ )
8. The height of 250 soldiers in a military camp confirms a normal distribution with mean height of 155 cms .and S.D. of 20 cms . Find the proportion of soldiers with height above 170 cms .
(Given, Area between 0 to 1.5 is 0.4332 and between 0 to 0.84 is 0.3 )
9. If the mean and variance of a binomial distribution are 4 and 2.4 respectively find the probability of (i) 8 success (ii) at least 9 successes
10. It is observed that $3 \%$ of apples in a consignment are bad. Find the probability that, in a consignment of 200 apples the number of bad apples is
(i) less than 2 (ii) only 3 . (Given that $\mathrm{e}^{-6}=0.00025$ )
11. The weights of 450 students in a school are normally distributed with the average weight of 50 kg . and S.D. 5 kg . Find the number of students with weight:
i) less than 45 kg . ii) Between 40 and 47 kg .
(Given: Area between 0 to 0.4 is 0.1554 , area between 0 to 0.5 is 0.1915 and area between 0 to 1 is 0.3413 .)
